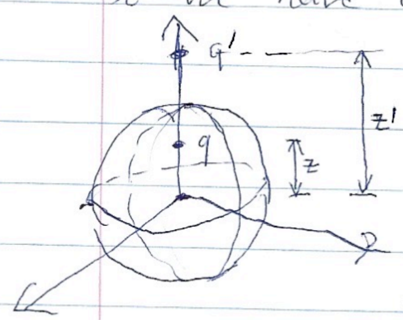
Jackson

2.2. (a). We produce the charge configuration by inverting the signs of that in section 2.2.

By spherical symmetry, we put the charge on the z-axis, so we have azimuthal (ϕ) symmetry.



Let q be inside the sphere with its coordinates given by $z\hat{z}$, and the image charge q' be at $z'\hat{z}$. We denote the position $\vec{x}$ by $x\hat{n}$.

$$\Phi(\vec{x}) = \frac{kq}{|x\hat{n} - z\hat{z}|} + \frac{kq'}{|x\hat{n} - z'\hat{z}|},$$

$$\Phi(|\vec{x}|=a) = \frac{kq}{a|\hat{n} - \frac{z}{a}\hat{z}|} + \frac{kq'}{z'|\hat{z} - \frac{a}{z'}\hat{n}|}$$

This can be satisfied by imposing $\frac{q}{a} = -\frac{q'}{z'}$, $\frac{a}{z'} = \frac{z}{a}$.

Thus $z' = a^2/z$, $q' = -\frac{a}{z}q$, giving potential

$$\boxed{\Phi(\vec{x}) = \frac{kq}{|x\hat{n} - z\hat{z}|} - \frac{kaq/z}{|x\hat{n} - \frac{a^2}{z}\hat{z}|}}$$

(b), (c) follow similarly as shown in Jackson section 2.2.

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(d). This is quite different from the case of charge ~~inside~~ outside a spherical conductor, because now we are interested in the inside of the conductor, thus we can't put a charge at the origin. Modification of part (a) gives the condition

$$\frac{kq}{a\left|\hat{n} - \frac{z}{a}\hat{z}\right|} + \frac{kq'}{z'\left|\hat{z} - \frac{a}{z'}\hat{n}\right|} = V.$$

Letting $\frac{V}{k} = \Omega / z'\left|\hat{z} - \frac{a}{z'}\hat{n}\right|$,

$$\frac{q}{a\left|\hat{n} - \frac{z}{a}\hat{z}\right|} = \frac{-q' + \Omega}{z'\left|\hat{z} - \frac{a}{z'}\hat{n}\right|} = \frac{-(q' - \Omega)}{z'\left|\hat{z} - \frac{a}{z'}\hat{n}\right|}.$$

Proceeding as we did for part (a), we obtain

$$\rightsquigarrow \frac{q}{a} = -\frac{(q' - \Omega)}{z'}, \quad \frac{z}{a} = \frac{a}{z'},$$

$$\rightsquigarrow z' = \frac{a^2}{z}, \quad q' = -\frac{a}{z}q + \Omega$$

$$= -\frac{a}{z}q + \frac{V}{k}\frac{a^2}{z}\left|\hat{z} - \frac{z}{a}\hat{n}\right|$$

This is unfortunate since it appears that the magnitude

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Jackson

2.2 ~~(b)~~ (d) Alternatively, by spherical symmetry, we put a charged spherical shell outside the conductor. By spherical symmetry, the potential on the conductor is constant.

To find the charge of the shell, we let it have radius $b > a$. We use the fact that the electric field inside spherical shell is zero. So the potential at the center of the shell must equal to everywhere inside the shell.

$$V(r=0) = k \int_V \frac{\rho}{|r'|} dr' = k \frac{Q}{4\pi b^2} \frac{1}{b} 4\pi b^2$$

$$= k \frac{Q}{b}$$

Then imposing $V = k\frac{Q}{b}$, $\boxed{Q = \frac{b}{k} V.}$

We find that constant potential can not be accomplished by a single mirror charge, we must use a charged shell as well then wield linear superposition.

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12.23.2023.